Simple Proof of a Fundamental Theorem in the Theory of Functions.

By R. D. Bohannan, *Ohio State University, Columbus, O.*

"If a Riemann's Surface is reduced by m cross-cuts into n distinct simply connected pieces, and by m' cross-cuts into n' such pieces, then $m - n = m' - n'$."

(a) In a simply connected surface, one cross-cut makes two simply connected pieces, and n cross-cuts make $n + 1$ such pieces.

(b) In a p-ply connected surface, $p - 1$ *appropriate* cross-cuts *are necessary* to reduce it to a simply connected surface.

(c) If a p-ply connected surface has been reduced by m cross-cuts to n simply connected pieces, the $p - 1$ cross-cuts noted in (b) have been made. Thus the n simply connected pieces are due to $m - (p - 1)$ of the cuts. Thus by (a),

$$n = m - (p - 1) + 1$$

$$\therefore\ m - n = p - 2, \text{ a constant for any given surface.}$$

Harkness and Morley, p. 229, and Forsyth, p. 317, give Neumann's proof. Riemann's proof will be found in his Gesammelte Werke, pp. 10, 11; also in Durège's Elemente der Theorie der Functionen, pp. 183–190. For Lippich's proof see Durège, pp. 190–197.